

# Generation of radiation in free electron lasers with diffraction gratings (photonic crystal) with the variable spacial period

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## Abstract

The equations providing to describe generation process in FEL with varied parameters of diffraction grating (photonic crystal) are obtained. It is shown that applying diffraction gratings (photonic crystal) with the variable period one can significantly increase radiation output. It is mentioned that diffraction gratings (photonic crystal) can be used for creation of the dynamical wiggler with variable period in the system. This makes possible to develop double-cascaded FEL with variable parameters changing, which efficiency can be significantly higher than of conventional system.

*Key words:* Free Electron Laser, travelling wave tube, backward wave oscillator, diffraction grating, Smith-Purcell radiation, diffraction radiation, photonic crystal

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## 1 Introduction

Generators using radiation from an electron beam in a periodic slow-wave circuit (travelling wave tubes, backward wave oscillators, free electron lasers) are now widespread [1].

Diffraction radiation [2] in periodical structures is in the basis of operation of travelling wave tubes (TWT) [3,4], backward wave oscillators (BWO) and such devices as Smith-Purcell lasers [5,6,7] and volume FELs using two- or three-dimensional distributed feedback [8,9,10,11].

Analysis shows that during operation of such devices electrons lose their energy for radiation, therefore, the electron beam slows down and gets out of synchronism with the radiating wave. These limits the efficiency of generator, which usually does not exceed  $\sim 10\%$ .

In the first years after creation of travelling wave tube it was demonstrated [4] that to retain synchronism between the electron beam and electromagnetic wave in a TWT change of the wave phase velocity should be provided. Application of systems with variable parameters in microwave devices allows significant increase of efficiency of such devices [4,12].

The same methods for efficiency increase are widely used for undulator FELs [13].

In the present paper we consider generation process in Smith-Purcell FELs, volume FELs, travelling wave tubes and backward wave oscillators using photonic crystal built from metal threads [14,15,16,17]. It is shown that applying diffraction gratings (photonic crystal) with the variable period one can sig-

nificantly increase radiation output. It is also shown that diffraction gratings (photonic crystal) can be used for creation of the dynamical wiggler with variable period in the system. This makes possible to develop double-cascaded FEL with variable parameters changing, which efficiency can be significantly higher than of conventional system.

## 2 Lasing equations for the system with a diffraction grating (photonic crystal) with changing parameters

In general case the equations, which describe lasing process, follow from the Maxwell equations:

$$\begin{aligned} \text{rot} \vec{H} &= \frac{1}{c} \frac{\partial \vec{D}}{\partial t} + \frac{4\pi}{c} \vec{j}, \quad \text{rot} \vec{E} = -\frac{1}{c} \frac{\partial \vec{H}}{\partial t}, \\ \text{div} \vec{D} &= 4\pi\rho, \quad \frac{\partial \rho}{\partial t} + \text{div} \vec{j} = 0, \end{aligned} \quad (1)$$

here  $\vec{E}$  and  $\vec{H}$  are the electric and magnetic fields,  $\vec{j}$  and  $\rho$  are the current and charge densities, the electromagnetic induction  $D_i(\vec{r}, t') = \int \varepsilon_{il}(\vec{r}, t - t') E_l(\vec{r}, t') dt'$  and, therefore,  $D_i(\vec{r}, \omega) = \varepsilon_{il}(\vec{r}, \omega) E_l(\vec{r}, \omega)$ , the indices  $i, l = 1, 2, 3$  correspond to the axes  $x, y, z$ , respectively.

The current and charge densities are respectively defined as:

$$\vec{j}(\vec{r}, t) = e \sum_{\alpha} \vec{v}_{\alpha}(t) \delta(\vec{r} - \vec{r}_{\alpha}(t)), \quad \rho(\vec{r}, t) = e \sum_{\alpha} \delta(\vec{r} - \vec{r}_{\alpha}(t)), \quad (2)$$

where  $e$  is the electron charge,  $\vec{v}_{\alpha}$  is the velocity of the particle  $\alpha$  ( $\alpha$  numerates the beam particles),

$$\frac{d\vec{v}_{\alpha}}{dt} = \frac{e}{m\gamma_{\alpha}} \left\{ \vec{E}(\vec{r}_{\alpha}(t), t) + \frac{1}{c} [\vec{v}_{\alpha}(t) \times \vec{H}(\vec{r}_{\alpha}(t), t)] - \frac{\vec{v}_{\alpha}}{c^2} (\vec{v}_{\alpha}(t) \cdot \vec{E}(\vec{r}_{\alpha}(t), t)) \right\}, \quad (3)$$

here  $\gamma_\alpha = (1 - \frac{v_\alpha^2}{c^2})^{-\frac{1}{2}}$  is the Lorentz-factor,  $\vec{E}(\vec{r}_\alpha(t), t)$  ( $\vec{H}(\vec{r}_\alpha(t), t)$ ) is the electric (magnetic) field in the point of location  $\vec{r}_\alpha$  of the particle  $\alpha$ . It should be reminded that the equation (3) can also be written as [?]:

$$\frac{d\vec{p}_\alpha}{dt} = m \frac{d\gamma_\alpha v_\alpha}{dt} = e \left\{ \vec{E}(\vec{r}_\alpha(t), t) + \frac{1}{c} [\vec{v}_\alpha(t) \times \vec{H}(\vec{r}_\alpha(t), t)] \right\}, \quad (4)$$

where  $p_\alpha$  is the particle momentum.

Combining the equations in (1) we obtain:

$$-\Delta \vec{E} + \vec{\nabla}(\vec{\nabla} \cdot \vec{E}) + \frac{1}{c^2} \frac{\partial^2 \vec{D}}{\partial t^2} = -\frac{4\pi}{c^2} \frac{\partial \vec{j}}{\partial t}. \quad (5)$$

The dielectric permittivity tensor can be expressed as  $\hat{\epsilon}(\vec{r}) = 1 + \hat{\chi}(\vec{r})$ , where  $\hat{\chi}(\vec{r})$  is the dielectric susceptibility. When  $\hat{\chi} \ll 1$  the equation (5) can be rewritten as:

$$\Delta \vec{E}(\vec{r}, t) - \frac{1}{c^2} \frac{\partial^2}{\partial t^2} \int \hat{\epsilon}(\vec{r}, t - t') \vec{E}(\vec{r}, t') dt' = 4\pi \left( \frac{1}{c^2} \frac{\partial \vec{j}(\vec{r}, t)}{\partial t} + \vec{\nabla} \rho(\vec{r}, t) \right). \quad (6)$$

When the grating is ideal  $\hat{\chi}(\vec{r}) = \sum_\tau \hat{\chi}_\tau(\vec{r}) e^{i\vec{\tau}\vec{r}}$ , where  $\vec{\tau}$  is the reciprocal lattice vector.

Let the diffraction grating (photonic crystal) period is smoothly varied with distance, which is much greater than the diffraction grating (photonic crystal lattice) period. It is convenient in this case to present the susceptibility  $\hat{\chi}(\vec{r})$  in the form, typical for theory of X-ray diffraction in crystals with lattice distortion [18]:

$$\hat{\chi}(\vec{r}) = \sum_\tau e^{i\Phi_\tau(\vec{r})} \hat{\chi}_\tau(\vec{r}), \quad (7)$$

where  $\Phi_\tau(\vec{r}) = \int \vec{\tau}(\vec{r}') d\vec{l}'$ ,  $\vec{\tau}(\vec{r}')$  is the reciprocal lattice vector in the vicinity

of the point  $\vec{r}'$ . In contrast to the theory of X-rays diffraction, in the case under consideration  $\hat{\chi}_\tau$  depends on  $\vec{r}$ . It is to the fact that  $\hat{\chi}_\tau$  depends on the volume of the lattice unit cell  $\Omega$ , which can be significantly varied for diffraction gratings (photonic crystals), as distinct from natural crystals. The volume of the unit cell  $\Omega(\vec{r})$  depends on coordinate and, for example, for a cubic lattice it is determined as  $\Omega(\vec{r}) = \frac{1}{d_1(\vec{r})d_2(\vec{r})d_3(\vec{r})}$ , where  $d_i$  are the lattice periods. If  $\hat{\chi}_\tau(\vec{r})$  does not depend on  $\vec{r}$ , the expression (7) converts to that usually used for X-rays in crystals with lattice distortion [18].

It should be reminded that for an ideal crystal without lattice distortions, the wave, which propagates in crystal can be presented as a superposition of the plane waves:

$$\vec{E}(\vec{r}, t) = \sum_{\vec{\tau}=0}^{\infty} \vec{A}_{\vec{\tau}} e^{i(\vec{k}_\tau \cdot \vec{r} - \omega t)}, \quad (8)$$

where  $\vec{k}_\tau = \vec{k} + \vec{\tau}$ .

Let us use now that in the case under consideration the typical length for change of the lattice parameters significantly exceeds lattice period. This provides to express the field inside the crystal with lattice distortion similarly (8), but with  $\vec{A}_{\vec{\tau}}$  depending on  $\vec{r}$  and  $t$  and noticeably changing at the distances much greater than the lattice period.

Similarly, the wave vector should be considered as a slowly changing function of coordinate.

According to the above let us find the solution of (6) in the form:

$$\vec{E}(\vec{r}, t) = \text{Re} \left\{ \sum_{\vec{\tau}=0}^{\infty} \vec{A}_{\vec{\tau}} e^{i(\phi_\tau(\vec{r}) - \omega t)} \right\}, \quad (9)$$

where  $\phi_\tau(\vec{r}) = \int_0^{\vec{r}} k(\vec{r}') d\vec{l}' + \Phi_\tau(\vec{r})$ , where  $k(\vec{r})$  can be found as solution of the dispersion equation in the vicinity of the point with the coordinate vector  $\vec{r}$ , integration is done over the quasiclassical trajectory, which describes motion of the wavepacket in the crystal with lattice distortion.

Let us consider now case when all the waves participating in the diffraction process lays in a plane (coupled wave diffraction, multiple-wave diffraction) i.e. all the reciprocal lattice vectors  $\vec{\tau}$  lie in one plane [21,20]. Suppose the wave polarization vector is orthogonal to the plane of diffraction.

Let us rewrite (9) in the form

$$\vec{E}(\vec{r}, t) = \vec{e} E(\vec{r}, t) = \vec{e} \operatorname{Re} \left\{ \vec{A}_1 e^{i(\phi_1(\vec{r}) - \omega t)} + \vec{A}_2 e^{i(\phi_2(\vec{r}) - \omega t)} + \dots \right\}, \quad (10)$$

where

$$\phi_1(\vec{r}) = \int_0^{\vec{r}} \vec{k}_1(\vec{r}') d\vec{l}', \quad (11)$$

$$\phi_2(\vec{r}) = \int_0^{\vec{r}} \vec{k}_1(\vec{r}') d\vec{l}' + \int_0^{\vec{r}} \vec{\tau}(\vec{r}') d\vec{l}'. \quad (12)$$

Then multiplying (6) by  $\vec{e}$  one can get:

$$\Delta E(\vec{r}, t) - \frac{1}{c^2} \frac{\partial^2}{\partial t^2} \int \hat{\varepsilon}(\vec{r}, t - t') E(\vec{r}, t') dt' = 4\pi \vec{e} \left( \frac{1}{c^2} \frac{\partial \vec{j}(\vec{r}, t)}{\partial t} + \vec{\nabla} \rho(\vec{r}, t) \right). \quad (13)$$

Applying the equality  $\Delta E(\vec{r}, t) = \vec{\nabla}(\vec{\nabla} E)$  and using (10) we obtain

$$\Delta(\vec{A}_1 e^{i(\phi_1(\vec{r}) - \omega t)}) = e^{i(\phi_1(\vec{r}) - \omega t)} [2i\vec{\nabla} \phi_1 \vec{\nabla} A_1 + i\vec{\nabla} \vec{k}_1(\vec{r}) A_1 - k_1^2(\vec{r}) A_1], \quad (14)$$

Therefore, substitution the above to (13) gives the following system:

$$\begin{aligned}
& \frac{1}{2} e^{i(\phi_1(\vec{r}) - \omega t)} [2i\vec{k}_1(\vec{r})\vec{\nabla}A_1 + i\vec{\nabla}\vec{k}_1(\vec{r})A_1 - k_1^2(\vec{r})A_1 + \\
& + \frac{\omega^2}{c^2}\varepsilon_0(\omega, \vec{r})A_1 + i\frac{1}{c^2}\frac{\partial\omega^2\varepsilon_0(\omega, \vec{r})}{\partial\omega}\frac{\partial A_1}{\partial t} + \frac{\omega^2}{c^2}\varepsilon_{-\tau}(\omega, \vec{r})A_2 + i\frac{1}{c^2}\frac{\partial\omega^2\varepsilon_{-\tau}(\omega, \vec{r})}{\partial\omega}\frac{\partial A_2}{\partial t}] + \\
& + \text{conjugated terms} = 4\pi\vec{e}\left(\frac{1}{c^2}\frac{\partial\vec{j}(\vec{r}, t)}{\partial t} + \vec{\nabla}\rho(\vec{r}, t)\right), \\
& \frac{1}{2} e^{i(\phi_2(\vec{r}) - \omega t)} [2i\vec{k}_2(\vec{r})\vec{\nabla}A_2 + i\vec{\nabla}\vec{k}_2(\vec{r})A_2 - k_2^2(\vec{r})A_2 + \\
& + \frac{\omega^2}{c^2}\varepsilon_0(\omega, \vec{r})A_2 + i\frac{1}{c^2}\frac{\partial\omega^2\varepsilon_0(\omega, \vec{r})}{\partial\omega}\frac{\partial A_2}{\partial t} + \frac{\omega^2}{c^2}\varepsilon_\tau(\omega, \vec{r})A_1 + i\frac{1}{c^2}\frac{\partial\omega^2\varepsilon_\tau(\omega, \vec{r})}{\partial\omega}\frac{\partial A_1}{\partial t}] + \\
& + \text{conjugated terms} = 4\pi\vec{e}\left(\frac{1}{c^2}\frac{\partial\vec{j}(\vec{r}, t)}{\partial t} + \vec{\nabla}\rho(\vec{r}, t)\right), \tag{15}
\end{aligned}$$

where the vector  $\vec{k}_2(\vec{r}) = \vec{k}_1(\vec{r}) + \vec{\tau}$ ,  $\varepsilon_0(\omega, \vec{r}) = 1 + \chi_0(\vec{r})$ , here notation  $\chi_0(\vec{r}) = \chi_{\tau=0}(\vec{r})$  is used,  $\varepsilon_\tau(\omega, \vec{r}) = \chi_\tau(\vec{r})$ . Note here that for numerical analysis of (15), if  $\chi_0 \ll 0$ , it is convenient to take the vector  $\vec{k}_1(\vec{r})$  in the form  $\vec{k}_1(\vec{r}) = \vec{n}\sqrt{k^2 + \frac{\omega^2}{c^2}\chi_0(\vec{r})}$ .

For better understanding let us suppose that the diffraction grating (photonic crystal lattice) period changes along one direction and define this direction as axis  $z$ .

Thus, for one-dimensional case, when  $\vec{k}(\vec{r}) = (\vec{k}_\perp, k_z(z))$  the system (15) converts to the following:

$$\begin{aligned}
& \frac{1}{2} e^{i(\vec{k}_\perp\vec{r}_\perp + \phi_{1z}(z) - \omega t)} [2ik_{1z}(z)\frac{\partial A_1}{\partial z} + i\frac{\partial k_{1z}(z)}{\partial z}A_1 - (k_\perp^2 + k_{1z}^2(z))A_1 + \\
& + \frac{\omega^2}{c^2}\varepsilon_0(\omega, z)A_1 + i\frac{1}{c^2}\frac{\partial\omega^2\varepsilon_0(\omega, z)}{\partial\omega}\frac{\partial A_1}{\partial t} + \frac{\omega^2}{c^2}\varepsilon_{-\tau}(\omega, z)A_2 + i\frac{1}{c^2}\frac{\partial\omega^2\varepsilon_{-\tau}(\omega, z)}{\partial\omega}\frac{\partial A_2}{\partial t}] + \\
& + \text{conjugated terms} = 4\pi\vec{e}\left(\frac{1}{c^2}\frac{\partial\vec{j}(\vec{r}, t)}{\partial t} + \vec{\nabla}\rho(\vec{r}, t)\right), \\
& \frac{1}{2} e^{i(\vec{k}_\perp\vec{r}_\perp + \phi_{2z}(z) - \omega t)} [2ik_{2z}(z)\frac{\partial A_2}{\partial z} + i\frac{\partial k_{2z}(z)}{\partial z}A_2 - (k_\perp^2 + k_{2z}^2(z))A_2 + \\
& + \frac{\omega^2}{c^2}\varepsilon_0(\omega, z)A_2 + i\frac{1}{c^2}\frac{\partial\omega^2\varepsilon_0(\omega, z)}{\partial\omega}\frac{\partial A_2}{\partial t} + \frac{\omega^2}{c^2}\varepsilon_\tau(\omega, z)A_1 + i\frac{1}{c^2}\frac{\partial\omega^2\varepsilon_\tau(\omega, z)}{\partial\omega}\frac{\partial A_1}{\partial t}] + \\
& + \text{conjugated terms} = 4\pi\vec{e}\left(\frac{1}{c^2}\frac{\partial\vec{j}(\vec{r}, t)}{\partial t} + \vec{\nabla}\rho(\vec{r}, t)\right), \tag{16}
\end{aligned}$$

Let us multiply the first equation by  $e^{-i(\vec{k}_\perp \vec{r}_\perp + \phi_{1z}(z) - \omega t)}$  and the second by  $e^{-i(\vec{k}_\perp \vec{r}_\perp + \phi_{2z}(z) - \omega t)}$ . This procedure provides to neglect the conjugated terms, which appear fast oscillating (when averaging over the oscillation period they become zero).

Considering the right part of (16) let us take into account that microscopic currents and densities are the sums of terms, containing delta-functions, therefore, the right part can be rewritten as:

$$\begin{aligned} & e^{-i(\vec{k}_\perp \vec{r}_\perp + \phi_{1z}(z) - \omega t)} 4\pi \vec{e} \left( \frac{1}{c^2} \frac{\partial \vec{j}(\vec{r}, t)}{\partial t} + \vec{\nabla} \rho(\vec{r}, t) \right) = \\ & = -\frac{4\pi i \omega e}{c^2} \vec{e} \sum_{\alpha} \vec{v}_{\alpha}(t) \delta(\vec{r} - \vec{r}_{\alpha}(t)) e^{-i(\vec{k}_\perp \vec{r}_\perp + \phi_{1z}(z) - \omega t)} \theta(t - t_{\alpha}) \theta(T_{\alpha} - t) \end{aligned} \quad (17)$$

here  $t_{\alpha}$  is the time of entrance of particle  $\alpha$  to the resonator,  $T_{\alpha}$  is the time of particle leaving from the resonator,  $\theta$ -functions in (ref5) image the fact that for time moments preceding  $t_{\alpha}$  and following  $T_{\alpha}$  the particle  $\alpha$  does not contribute in process.

Let us suppose now that a strong magnetic field is applied for beam guiding though the generation area. Thus, the problem appears one-dimensional (components  $v_x$  and  $v_y$  are suppressed). Averaging the right part of (18) over the particle positions inside the beam, points of particle entrance to the resonator  $r_{\perp 0\alpha}$  and time of particle entrance to the resonator  $t_{\alpha}$  we can obtain:

$$\begin{aligned} & e^{-i(\vec{k}_\perp \vec{r}_\perp + \phi_{1z}(z) - \omega t)} 4\pi \vec{e} \left( \frac{1}{c^2} \frac{\partial \vec{j}(\vec{r}, t)}{\partial t} + \vec{\nabla} \rho(\vec{r}, t) \right) = \\ & = -\frac{4\pi i \omega \rho \vartheta_1 u(t) e}{c^2} \frac{1}{S} \int d^2 \vec{r}_{\perp 0} \frac{1}{T} \int_0^t e^{-i(\phi_1(\vec{r}, \vec{r}_{\perp}, t, t_0) + \vec{k}_\perp \vec{r}_{\perp 0} - \omega t)} dt_0 = \\ & = -\frac{4\pi i \omega \rho \vartheta_1 u(t) e}{c^2} << e^{-i(\phi_1(\vec{r}, \vec{r}_{\perp}, t, t_0) + \vec{k}_\perp \vec{r}_{\perp 0} - \omega t)} dt_0 >>, \end{aligned} \quad (18)$$

where  $\rho$  is the electron beam density ,  $u(t)$  is the mean electron beam velocity,

which depends on time due to energy losses,  $\vartheta_1 = \sqrt{1 - \frac{\omega^2}{\beta^2 k_1^2 c^2}}$ ,  $\beta^2 = 1 - \frac{1}{\gamma^2}$ ,  $\langle \langle \dots \rangle \rangle$  indicates averaging over transversal coordinate of point of particle entrance to the resonator  $r_{\perp 0\alpha}$  and time of particle entrance to the resonator  $t_\alpha$ .

According to [22] averaging procedure in (18) can be simplified, when consider that random phases, appearing due to random transversal coordinate and time of entrance, presents in (18) as differences. Therefore, double integration over  $d^2 \vec{r}_{\perp 0} dt_0$  can be replaced by single integration [22].

The system (16) in this case converts to:

$$\begin{aligned} & 2ik_{1z}(z) \frac{\partial A_1}{\partial z} + i \frac{\partial k_{1z}(z)}{\partial z} A_1 - (k_\perp^2 + k_{1z}^2(z)) A_1 + \\ & + \frac{\omega^2}{c^2} \varepsilon_0(\omega, z) A_1 + i \frac{1}{c^2} \frac{\partial \omega^2 \varepsilon_0(\omega, z)}{\partial \omega} \frac{\partial A_1}{\partial t} + \frac{\omega^2}{c^2} \varepsilon_{-\tau}(\omega, z) A_2 + \\ & + i \frac{1}{c^2} \frac{\partial \omega^2 \varepsilon_{-\tau}(\omega, z)}{\partial \omega} \frac{\partial A_2}{\partial t} = i \frac{2\omega}{c^2} J_1(k_{1z}(z)), \\ & 2ik_{2z}(z) \frac{\partial A_2}{\partial z} + i \frac{\partial k_{2z}(z)}{\partial z} A_2 - (k_\perp^2 + k_{2z}^2(z)) A_2 + \\ & + \frac{\omega^2}{c^2} \varepsilon_0(\omega, z) A_2 + i \frac{1}{c^2} \frac{\partial \omega^2 \varepsilon_0(\omega, z)}{\partial \omega} \frac{\partial A_2}{\partial t} + \frac{\omega^2}{c^2} \varepsilon_\tau(\omega, z) A_1 + \\ & + i \frac{1}{c^2} \frac{\partial \omega^2 \varepsilon_\tau(\omega, z)}{\partial \omega} \frac{\partial A_1}{\partial t} = i \frac{2\omega}{c^2} J_2(k_{2z}(z)), \end{aligned} \quad (19)$$

where the currents  $J_1$ ,  $J_2$  are determined by the expression

$$J_m = 2\pi j \vartheta_m \int_0^{2\pi} \frac{2\pi - p}{8\pi^2} (e^{-i\phi_m(t, z, p)} + e^{-i\phi_m(t, z, -p)}) dp, \quad m = 1, 2 \quad (20)$$

$$\vartheta_m = \sqrt{1 - \frac{\omega^2}{\beta^2 k_m^2 c^2}}, \quad \beta^2 = 1 - \frac{1}{\gamma^2},$$

$j = en_0 v$  is the current density,  $A_1 \equiv A_{\tau=0}$ ,  $A_2 \equiv A_\tau$ ,  $\vec{k}_1 = \vec{k}_{\tau=0}$ ,  $\vec{k}_2 = \vec{k}_1 + \vec{\tau}$ .

The expressions for  $J_1$  for  $k_1$  independent on  $z$  was obtained in [22].

When more than two waves participate in diffraction process, the system (20)

should be supplemented with equations for waves  $A_m$ , which are similar to those for  $A_1$  and  $A_2$ .

Now we can find the equation for phase. From the expressions (11,12) it follows that

$$\frac{d^2\phi_m}{dz^2} + \frac{1}{v} \frac{dv}{dz} \frac{d\phi_m}{dz} = \frac{dk_m}{dz} + \frac{k_m}{v^2} \frac{d^2z}{dt^2}, \quad (21)$$

Let us introduce new function  $C(z)$  as follows:

$$\begin{aligned} \frac{d\phi_m}{dz} &= C_m(z) e^{-\int_0^z \frac{1}{v} \frac{dv}{dz'} dz'} = \frac{v_0}{v(z)} C_m(z), \\ \phi_m(z) &= \phi_m(0) + \int_0^z \frac{v_0}{v(z')} C_m(z') dz' \end{aligned} \quad (22)$$

Therefore,

$$\frac{dC_m(z)}{dz} = \frac{v(z)}{v_0} \left( \frac{dk_m}{dz} + \frac{k_m}{v^2} \frac{d^2z}{dt^2} \right). \quad (23)$$

In the one-dimensional case the equation (4) can be written as:

$$\frac{d^2z_\alpha}{dt^2} = \frac{e\vartheta}{m\gamma(z_\alpha, t, p)} \operatorname{Re} E(z_\alpha, t), \quad (24)$$

therefore,

$$\frac{dC_m(z)}{dz} = \frac{v(z)}{v_0} \frac{dk_m}{dz} + \frac{k_m}{v_0 v(z)} \frac{e\vartheta_m}{m\gamma^3(z, t(z), p)} \operatorname{Re} \{ A_m(z, t(z)) e^{i\phi_m(z, t(z), p)} \}, \quad (25)$$

$$\frac{d\phi_m(t, z, p)}{dz}|_{z=0} = k_{mz} - \frac{\omega}{v}, \quad \phi_m(t, z, p)|_{z=0} = p,$$

$$A_1|_{z=L} = E_1^0, \quad A_2|_{z=L} = E_2^0,$$

$$A_m|_{t=0} = 0, \quad m = 1, 2,$$

$t > 0$ ,  $z \in [0, L]$ ,  $p \in [-2\pi, 2\pi]$ ,  $L$  is the length of the photonic crystal.

These equations should be supplied with the equations for  $\gamma(z, p)$ . It is well-known that

$$mc^2 \frac{d\gamma}{dt} = e\vec{v}\vec{E}. \quad (26)$$

Therefore,

$$\frac{d\gamma(z, t(z), p)}{dz} = \sum_l \frac{e\vartheta_l}{mc^2} \operatorname{Re}\left\{\sum_l A_l(z, t(z)) e^{i\phi_l(z, t(z), p)}\right\}. \quad (27)$$

The above obtained equations (20,23,25,27) provide to describe generation process in FEL with varied parameters of diffraction grating (photonic crystal). Analysis of the system (25) can be simplified by replacement of the  $\gamma(z, t(z), p)$  with its averaged by the initial phase value

$$\langle \gamma(z, t(z)) \rangle = \frac{1}{2\pi} \int_0^{2\pi} \gamma(z, t(z), p) dp.$$

Note that the law of parameters change can be both smooth and stair-step.

Use of photonic crystals provide to develop different VFEL arrangements (see Fig.1).

It should be noted that, for example, in the FEL (TWT,BWO) resonator with changing in space parameters of grating (photonic crystal) the electromagnetic wave with depending on  $z$  spatial period is formed. This means that the dynamical undulator with depending on  $z$  period appears along the whole resonator length i. e. tapering dynamical wiggler becomes settled. It is well known that tapering wiggler can significantly increase efficiency of the undulator FEL. The dynamical wiggler with varied period, which is proposed, can

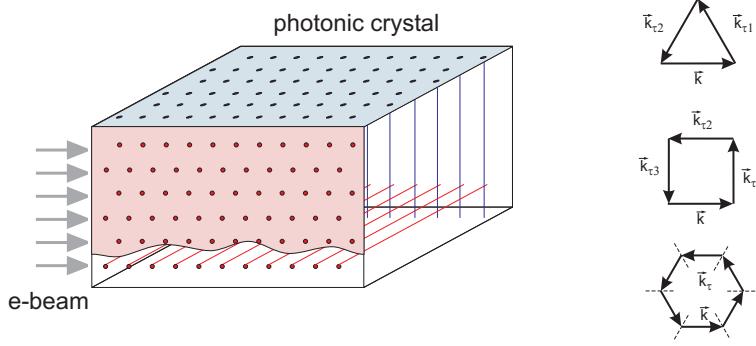


Fig. 1. An example of photonic crystal with the thread arrangement providing multi-wave volume distributed feedback. Threads are arranged to couple several waves (three, four, six and so on), which appear due to diffraction in such a structure, in both the vertical and horizontal planes. The electronic beam takes the whole volume of photonic crystal.

be used for development of double-cascaded FEL with parameters changing in space. The efficiency of such system can be significantly higher than of conventional system. Moreover, the period of dynamical wiggler can be done much shorter than that available for wigglers using static magnetic fields. It should be also noted that, due to dependence of the phase velocity of the electromagnetic wave on time, compression of the radiation pulse is possible in such a system.

### 3 Conclusion

The equations providing to describe generation process in FEL with varied parameters of diffraction grating (photonic crystal) are obtained. It is shown that applying diffraction gratings (photonic crystal) with the variable period one can significantly increase radiation output. It is mentioned that diffraction gratings (photonic crystal) can be used for creation of the dynamical

wiggler with variable period in the system. This makes possible to develop double-cascaded FEL with variable parameters changing, which efficiency can be significantly higher than that of conventional system.

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